

# Forecast congruence, accuracy, and smoothing parameter shrinkage

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Marketing Analytics  
and Forecasting



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# Forecasting and decisions

Managers use forecasts to make decisions:

- Producing forecasts;
- Making decisions based on forecasts.

It is often difficult to merge these activities into a single activity.

The literature argues that accurate forecasts lead to desirable decisions.

However, this might not always be the case.

# Forecasting and decisions

Pritularga and Kourentzes (2024) propose a new measure to connect forecasting with inventory decisions – **forecast congruence**

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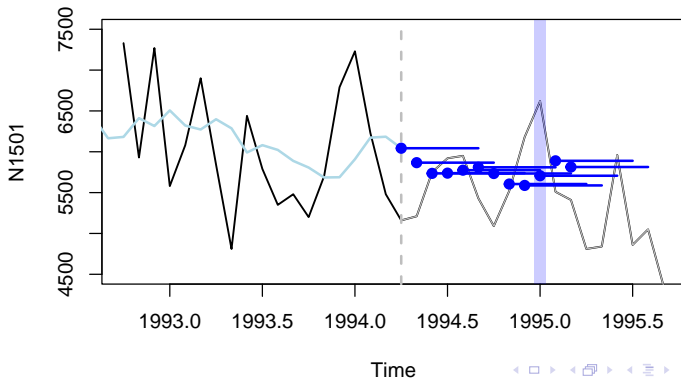
Let's forecast sales of double espresso tomorrow.

We produce many forecasts from many origins, e.g., from the last week to yesterday; from different step ahead forecasts.

The paper argues that we need to get congruent forecasts that do not harm accuracy.

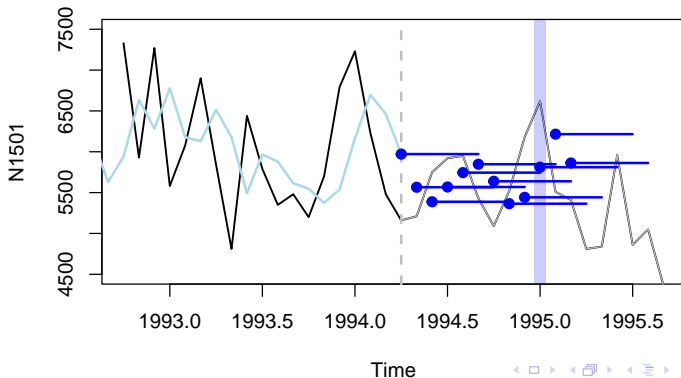
# Forecasting and decisions

ETS(A,N,N) with  $\alpha = 0.2$



# Forecasting and decisions

ETS(A,N,N) with  $\alpha = 0.5$



# Forecasting and decisions

How can we achieve this?

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- Do not re-estimate the forecasting models across origins (ad-hoc approach);
- Multiple temporal aggregation approach (e.g., MAPA, THieF, (Kourentzes et al., 2014; Athanasopoulos et al., 2017));
- Exponential smoothing (ETS) with shrinkage estimators.
- Machine learning approach (Van Belle et al., 2023).

Changing the loss function to produce ‘more stable’ or congruent forecasts (Van Belle et al., 2023; Godahewa et al., 2023).



# ETS and decisions

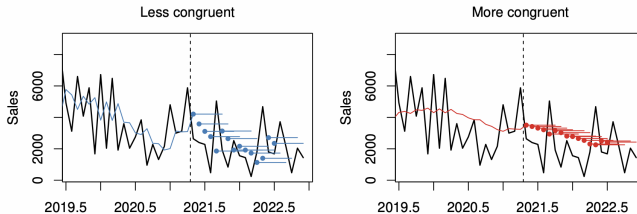
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# ETS and decisions

ETS shrinkage offers a way to control the value of the smoothing parameters.

This then makes the states (level, trend, or seasons) tend to be less stochastic.

Then, the forecasts tend to be more congruent.



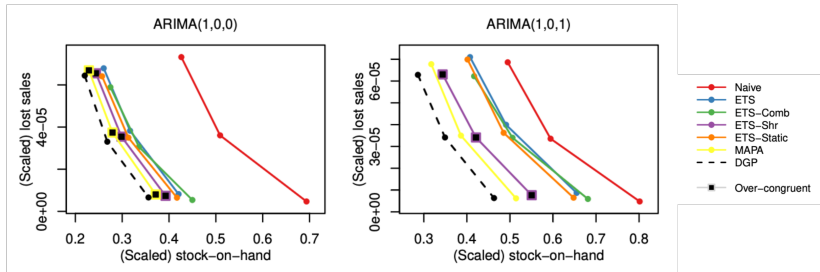
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This leads to less desirable inventory decision metrics such as lost sales and service levels.



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From here forecast congruence is our proxy for decisions



# ETS and forecast congruence

Let's take a simple example of ETS(ANN)

$$y_t = l_{t-1} + \varepsilon_t$$

$$l_t = l_{t-1} + \alpha \varepsilon_t$$

where  $\alpha$  is the smoothing parameter and it controls the stochasticity of the local mean.  $l_0$  is the level initial value and it initialises the time series.

We need to estimate the parameters of  $\alpha$  and  $l_0$  with any estimators (MSE, MAE, MLE, etc.).

# ETS and forecast congruence

The available shrinkage estimators implement 'LASSO' and 'RIDGE'-like estimators

$$\text{LASSO} = (1 - \lambda)\text{MSE}_{t+1|t} + \lambda \sum_{j=1}^K |g_i| \quad (1)$$

$$\text{RIDGE} = (1 - \lambda)\text{MSE}_{t+1|t} + \lambda \sum_{j=1}^K g_i^2 \quad (2)$$

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where  $g_i \in \mathbf{g}$ ,  $\lambda$  is a hyperparameter and  $\lambda$  is between 0 and 1.

This means that we shrink the smoothing parameters ( $g_i$ ) to zero and putting the importance of the initial values in ETS. Note: the dampening parameter is shrunk to 1.

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The current implementation relies on the one-step ahead MSE in the validation set:

$$\mathcal{L}(\lambda, \hat{\mathbf{g}}) = \min_{\lambda} \frac{1}{L} \sum_{l=1}^L \text{MSE}(\hat{y}_{t+1} | \mathcal{I}_{t,l}, \hat{\mathbf{g}})$$

where  $L$  is the number of forecast origins and  $\mathcal{I}_{t,l}$  is the information at time  $t$  and origin  $l$ .

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where  $L$  is the number of forecast origins and  $\mathcal{I}_{t,l}$  is the information at time  $t$  and origin  $l$ . What does it mean?

# ETS and forecast congruence

This means that  $\lambda$  is optimised for a very short forecast horizon.



# ETS and forecast congruence

This means that  $\lambda$  is optimised for a very short forecast horizon.

This does not align well with the nature of forecast congruence – **multistep forecast errors**.

$$\begin{pmatrix}
 e_{1+l|l} & e_{1+l|l-1} & \cdots & e_{1+l|1+l-(l-1)} & e_{1+l|1} \\
 e_{2+l|1+l} & e_{2+l|l} & \cdots & e_{2+l|2+l-(l-1)} & e_{1+l|2} \\
 \vdots & \ddots & & & \vdots \\
 & & e_{t|t-h} & & \vdots \\
 \vdots & & & \ddots & \vdots \\
 e_{n-1|n-2} & e_{n-1|n-3} & \cdots & e_{n-1|n-1-(l-1)} & e_{n-1|n-1-l} \\
 e_{n|n-1} & e_{n|n-2} & \cdots & e_{n|n-(l-1)} & e_{n|n-l}
 \end{pmatrix}
 \begin{matrix}
 \rightarrow \tau_{l+1}^2 \\
 \rightarrow \tau_{l+2}^2 \\
 \\
 \vdots \\
 \rightarrow \tau_{n-1}^2 \\
 \rightarrow \tau_n^2
 \end{matrix}$$

$$\begin{matrix}
 \downarrow & \downarrow & & \downarrow & \downarrow \\
 \sigma_1^2 & \sigma_2^2 & \cdots & \sigma_{l-1}^2 & \sigma_l^2
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Steps in answering this question:

1. exploring the relationship between accuracy (MSE) and congruence ( $\tau$ ), conditional to different circumstances;
2. using the exploration findings to propose a new loss function for finding the hyperparameter.

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Let's define forecast congruence. We are interested in collecting a set of forecasts from different origins and  $h$  for a specific  $t$ .

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$$\begin{aligned}\tau &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(\hat{y}_{t|t-h})} \right) \\ &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(y_t - e_{t|t-h})} \right) \\ \tau &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(e_{t|t-h})} \right)\end{aligned}$$

$\tau$  measures the variance of  $e_{t|t-h}$  over  $h$

# Accuracy & congruence: analytical approach

Random walk

$$\tau_t = \sqrt{\mathbb{E} \left[ \sum_{i=1}^h (h-i+1) \varepsilon_{t-i+1}^2 \right] - \mathbb{E} \left[ \sum_{i=1}^h (h-i+1) \varepsilon_{t-i+1} \right]^2},$$

AR(1)

$$\tau_t = \sqrt{\mathbb{E} \left[ \sum_{i=1}^h (h-i+1) \alpha^{2i} \varepsilon_{t-i+1}^2 \right] - \mathbb{E} \left[ \sum_{i=1}^h (h-i+1) \alpha^i \varepsilon_{t-i+1} \right]^2},$$

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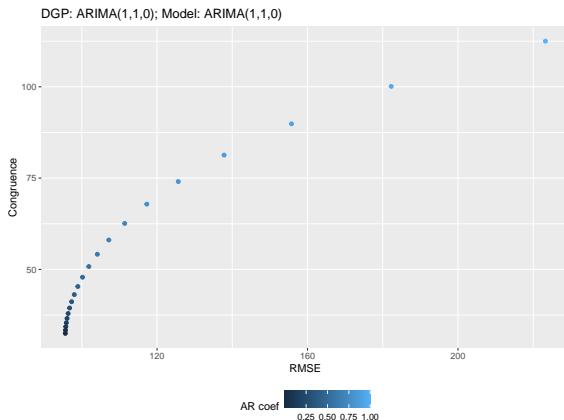
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The positive relationship between  $\tau$  and  $\sigma$  is not only mediated by the horizon but also the model parameters.

# Accuracy & congruence: simulation approach

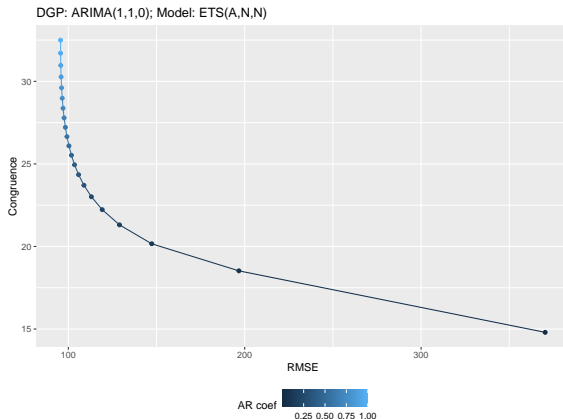
Correct models, 1000 repetitions





# Accuracy & congruence: simulation approach

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# Multistep estimators

We need a metric that takes into account the ‘multi-step’ nature –  
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Literature has studied the efficacy of multistep loss function in forecasting (Svetunkov et al., 2024, and the reference therein).

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Minimising h-step ahead forecast variance eventually shrinks the coefficients of SSOE-based models.

$GMSE_h$  is the geometric mean of  $\log MSE_j$  for  $j \in h$  - with a slight modification.

$$GMSE_h = \frac{1}{h} \sum_{j=1}^h \log \frac{1}{T-h} \sum_{t=1}^{T-h} e_{t+j|t}^2 = \frac{1}{h} \sum_{j=1}^h \log MSE_j$$

# A composite loss: multistep accuracy + congruence

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How do we find the 'optimal'  $\lambda$ ? Note:  $0 \leq \lambda < 1$

1. Split the dataset into (a) training and (b) validation set.
2. Estimate ETS parameters with a composite loss function for each  $\lambda$ .
3. Measure the composite loss in the validation set.
4. Find  $\lambda$  that results in the lowest composite loss.

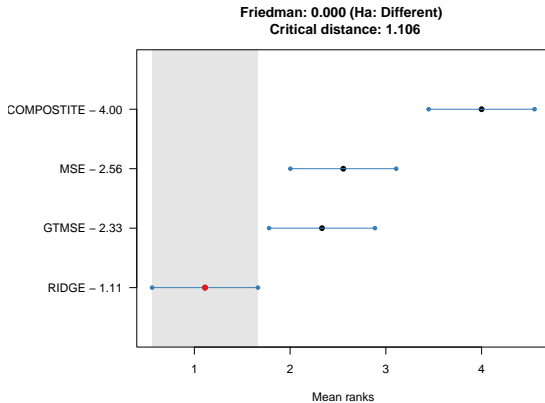
# Empirical analysis

- Dataset: M3 dataset for monthly only.
- Split the training set into an in-sample and a validation set (6 origins).
- The model structure is set to be incorrect: ETS(M,A,M)
- Forecast horizon: 18.
- MSE as the benchmark.
- Evaluated the point forecast accuracy for 1-h step ahead in the test set using RMSSE and scaled AME.
- Include  $\tau$  that is calculated in the validation set.



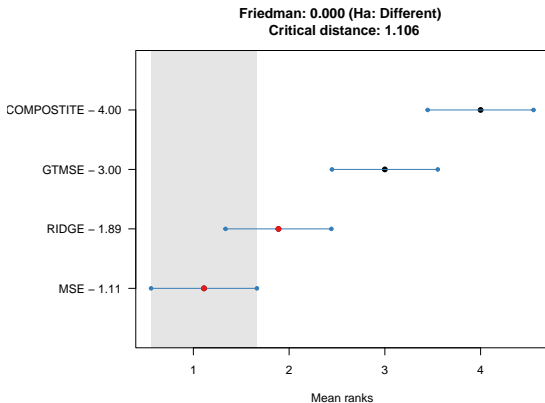
# Empirical analysis

Accuracy – MSE (across hi



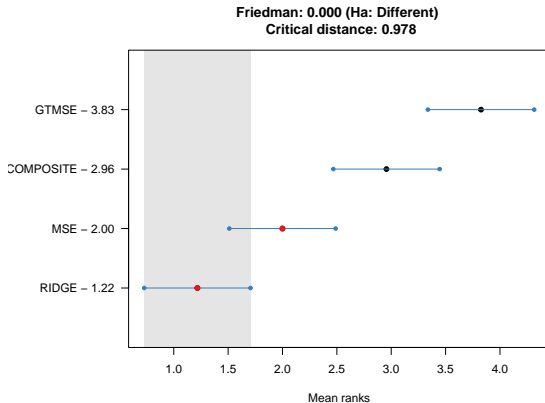
# Empirical analysis

## Bias – AME



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## Congruence – $\tau$



## Discussion & conclusion

RIDGE and MSE perform well for accuracy, bias, and congruence when the model structure is incorrectly estimated.

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This experiment is motivated by the fact that we often have to estimate an incorrect model structure.

The experiment design could have been improved by providing the case when the model structure is estimated via an information criterion.

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The experiment design could have been improved by providing the case when the model structure is estimated via an information criterion.

The data-driven approach to find  $\lambda$  is not suitable in this particular situation.

## Discussion & conclusion

Congruence offers a theoretical understanding of the ‘good’ forecasts, where forecasts agree with each other.

Forecasting models/ methods that have the smoothing feature on them:

- MAPA;
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However, more questions than answers...



## Discussion & conclusion

The congruence measures the balance between the column-wise and the row-wise of the forecast error matrix.

- Can we create a 'seemingly-like' covariance matrix of the multistep errors that contains accuracy and congruence?
  - ▶ a multistep error matrix has a dimension of  $n \times h$ , where  $n$  is the number of the origins and  $h$  is the forecast horizon
  - ▶ the covariance matrix of the multistep error will have a dimension of  $n \times n$
- But then  $\tau$  can have a matrix of  $h \times h$  – does  $\tau$  have another version of the covariance matrix?

Is  $\lambda$  too restrictive? Should we have a different way of formulating the hyperparameter loss function?

# Why not forecast stability?

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Nonetheless, we consider a few alternatives until we find a ‘free’  
word.

Congruence here means ‘agreement’ between forecasts, due to  
updating information (re-estimate each model on each origin).

# Thank you for your attention!

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## Forecast congruence

$e_{t|t-h}$  contains the innovation at time  $t$  ( $\varepsilon_t$ ) and the ‘other’ errors  $\eta_{t|t-h}$  due to incorrect model structures and parameters.

$$\begin{aligned}\tau &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(\hat{y}_{t|t-h})} \right) \\ &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(y_t - e_{t|t-h})} \right) \\ &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(\eta_{t|t-h} + \varepsilon_t)} \right) \\ &= \frac{1}{n} \sum_{t=1}^n \left( \sqrt{\text{var}_h(\eta_{t|t-h})} \right)\end{aligned}$$

$\tau$  eventually measures the variance of misspesification errors 